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PLANAR PROBLEM OF HYDRODYNAMIC SHAKING OF A SUBMERGED BODY IN THE PRESENCE OF MOTION IN A TWO-LAYERED FLUID

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We investigate the shaking of a regularly agitated horizontal cylindrical body, located in the lower layer of a two-layered fluid. The body moves uniformly at a constant depth in the direction perpendicular to its axis, and carries out harmonic oscillations under the action of parallel or opposite waves. The given problem is solved by linear theory methods. The free waves appearing on the body, as well as the induced shaking of the submerged body and the waves thus formed, are assumed to be small. The fluid is assumed to be ideal and incompressible, with potential flow in each layer.

The presently available theoretical studies of hydrodynamic shaking of a submerged body in the presence of motion [1-8] were carried out only for a homogeneous fluid, and with the single source of external agitation being surface waves. A more complex problem is the study of the effect of external agitation on a body moving in a stratified fluid. In this case the agitation can be generated by both surface and free internal waves. The motion of a body under the free surface of a homogeneous fluid is a special case of this problem.

1. Statement of the Problem. In the unperturbed state the upper layer of the fluid of density ρ_1 has a depth H and occupies the region $-\infty < \bar{x} < \infty$, $0 < y < H$, while the lower infinitely deep fluid layer of density $\rho_2 = (1 + \varepsilon)\rho_1$ ($\varepsilon > 0$) occupies the region $-\infty < \bar{x} < \infty$, $y < 0$, where $\bar{x}$ is the horizontal and y is the vertical coordinate. Along with the fixed coordinate system $(\bar{x}, y)$ we introduce the moving system $(x = \bar{x} - Ut, y)$, moving together with the body with constant velocity U. For simplicity it is assumed that the upper layer of the fluid is bounded by a solid cover. The existence of only free internal waves, resulting from the presence of a separation boundary between the layers, is possible. In the fixed coordinate system the velocity potential for the incident internal wave is

$$\Psi_0^{(s)} = \frac{i\omega_0}{k_0}\varphi_0^{(s)}\exp[i(\omega_0 t \mp k_0\bar{x})],\ \varphi_0^{(1)} = -\frac{\mathrm{ch}k_0(y - H)}{\mathrm{sh}k_0H},\ \varphi_0^{(s)} = e^{k_0 y}, \tag{1.1}$$

where only the real part has a physical meaning; ω_0 is the wave frequency in the fixed coordinate system, the plus and minus signs correspond to the codirectional and opposite waves, the wave number of the incident waves k_0 is determined from the dispersion relation

$$\omega_0^2 = \Omega^2(k_0) \equiv \varepsilon g k_0/(1 + \varepsilon + \mathrm{cth}k_0H); \tag{1.2}$$

the indices s = 1, 2 were introduced for the upper and lower layers, and g is the gravity field acceleration.

In the fixed coordinate system expression (1.1) acquires the form

$$\Psi_0^{(s)} = \frac{i\omega_0}{k_0}\varphi_0^{(s)}\exp[i(\omega t \mp k_0 x)].$$

Here ω is the apparent oscillation frequency of wave particles, which by the Doppler shift equals $\omega = \omega_0 \mp k_0U$.

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Assuming that the perturbed oscillatory motion of the fluid is steady-state, the total velocity potential of the whole wave motion is written in the form

$$\Phi^{(s)}(x,y,t) = -Ux + U\overline{\Phi}^{(s)}(x,y) + \operatorname{Re}\sum_{j=0}\eta_j\Phi_j^{(s)}(x,y)e^{i\omega t}, \tag{1.3}$$

where $\bar{\Phi}^{(s)}$ are the velocity potentials corresponding to uniform motion of a body with unit velocity, $\Phi_j^{(s)}$ ($j = \overline{1, 3}$) characterize the radiation potentials resulting from purely induced shaking of the body with three degrees of freedom in uniform fluid flow in the absence of incident waves, η_j are the amplitudes of body motion corresponding to the horizontal, vertical, and rotational body oscillations, $\Phi_0^{(s)} = \varphi_0^{(s)}\exp(\mp ik_0x)$. is the velocity potential of the incident wave, $\Phi_4^{(s)}$ are the diffraction potentials determining the wave motions generated as a result of incidence of regular waves on the body as a fixed obstacle, and $\eta_0 = \eta_4$ is the amplitude of the incident wave.

For the stationary potential inside the fluid we have

$$\Delta\overline{\Phi}^{(1)} = 0\ (0 < y < H),\ \Delta\overline{\Phi}^{(2)} = 0\ (y < 0). \tag{1.4}$$

Starting from linear wave theory, we write down the boundary conditions at the upper boundary

$$\partial\overline{\Phi}^{(1)}/\partial y = 0 \quad (y = H),$$

at the separation surface

$$(1+\varepsilon)\frac{\partial^2\overline{\Phi}^{(2)}}{\partial x^2} - \frac{\partial^2\overline{\Phi}^{(1)}}{\partial x^2} + \frac{\varepsilon g}{U^2}\frac{\partial\overline{\Phi}^{(1)}}{\partial y} = 0,\ \frac{\partial\overline{\Phi}^{(1)}}{\partial y} = \frac{\partial\overline{\Phi}^{(2)}}{\partial y} \quad (y = 0),$$

and in the far field

$$\frac{\partial\overline{\Phi}^{(2)}}{\partial y} \to 0\ (y \to -\infty),\ \frac{\partial\overline{\Phi}^{(s)}}{\partial x} \to 0\ (x \to \infty),\ \left|\frac{\partial\overline{\Phi}^{(s)}}{\partial x}\right| < \infty\ (x \to -\infty).$$

It is assumed that the body is completely located in the lower layer, and due to the smallness of its oscillations the nonleaking condition is imposed on the surface of the body in its mean position $L{:}\partial\overline{\Phi}^{(2)}/\partial n = n_x(x, y, \in L)$, where $n = (n_x, n_y)$ is the internal normal to the surface of the body.

Similarly to Eq. (1.4), the components of the radiation and diffraction potentials satisfy the equations

$$\Delta\overline{\Phi}_j^{(1)} = 0\ (0 < y < H),\ \Delta\Phi_j^{(2)} = 0\ (y < 0)$$

with boundary conditions:

$$\partial\Phi_j^{(1)}/\partial y = 0 \quad (y = H); \tag{1.5}$$

$$(1+\varepsilon)N\Phi_j^{(2)} - N\Phi_j^{(1)} + \varepsilon g\partial\Phi_j^{(1)}/\partial y = 0,\ \partial\Phi_j^{(1)}/\partial y = \partial\Phi_j^{(2)}/\partial y \quad (y = 0); \tag{1.6}$$

$$\partial\Phi_j^{(2)}/\partial y \to 0\ (y \to -\infty); \tag{1.7}$$

$$\partial\Phi_j^{(2)}/\partial n = i\omega n_j - Um_j\ (j = \overline{1,3}),\ \partial\Phi_4^{(2)}/\partial n = -\partial\Phi_0^{(2)}/\partial n \quad (x,y \in L). \tag{1.8}$$

Here

$$N \equiv (U\partial/\partial x - i\omega)^2;\ (n_1,n_2) = (n_x,n_y);\ n_3 = (y - y_0)n_x - (x - x_0)n_y;$$

$$(m_1,m_2,m_3) = \left\{\frac{\partial^2\overline{\Phi}^{(2)}}{\partial n\partial x}, \frac{\partial^2\overline{\Phi}^{(2)}}{\partial n\partial y}, \frac{\partial}{\partial n}\left[(y - y_0)\left(\frac{\partial\overline{\Phi}^{(2)}}{\partial x} - 1\right) - (x - x_0)\frac{\partial\overline{\Phi}^{(2)}}{\partial y}\right]\right\};$$

and x_0, y_0 are the coordinates of the point, with respect to which rotational oscillations of the body are carried out. The boundary conditions for $\Phi_j^{(s)}$ that a wave propagating ahead of the body can exist only when its phase velocity is positive, and the group velocity is higher than the body velocity, while in the opposite case wave motions exist only behind the body.

We note that in several studies (see, for example, [9]) the first boundary condition in (1.8) is given without the second term. The appearance of this term is related to taking into account the interaction between the stationary and oscillatory flow modes, i.e., taking into account the small deviation of the real body position from its mean position. The important role of this term in determining the hydrodynamic load, particularly at low frequencies, was investigated in detail in [5] for the example of a circular cylinder.

Hydrodynamic forces and torques, resulting from the body's translational motion, oscillations, and scattering waves, act on the body during the agitating motion. The general expressions for the force **F** and torque M are determined by integration of the fluid pressure $p = -\rho_2(\partial\Phi^{(2)}/\partial t + |\nabla\Phi^{(2)}|^2/2)$. over the surface of the body L:

$$\mathrm{F} = \int_L pn\,dl,\ M = \int_L pn_3\,dl. \tag{1.9}$$

It is convenient to introduce the representations $\mathbf{F} = (F_1, F_2)$, $M = F_3$ and replace (1.9) by the sum

$$F_j = F_{sj} + \mathrm{Re}[(F_{nj} + F_{ej})e^{i\omega t}], \tag{1.10}$$

where the first term results from the uniform translational motion of the body:

$$F_{sj} = \rho_2 U^2 \int_L (\partial\overline{\Phi}^{(2)}/\partial x - |\nabla\overline{\Phi}^{(2)}|^2/2)n_j\,dl;$$

and the second term is the contribution of the nonstationary potentials $\Phi_j^{(2)}$ $(j = \overline{1, 3})$. The three components of this force and the torque can be written in matrix form

$$F_{nj} = \sum_{k=1}^{3} \eta_k \tau_{jk},\ \tau_{jk} = -\rho_2 \int_L (i\omega\Phi_k^{(2)} + V\nabla\Phi_k^{(2)})n_j\,dl \tag{1.11}$$

($V = U\nabla(\overline{\Phi}^{(2)} - x)$ is the velocity vector of stationary flow in the lower layer relatively to the moving coordinate system). The coefficients τ_{jk} represent the complex force acting in direction j and resulting from sinusoidal body oscillations with unity amplitude in direction k. These coefficients can be represented in the form $\tau_{jk} = \omega^2\mu_{jk} - i\omega\lambda_{jk}$, where μ_{jk} are known as combined mass coefficients, and λ_{jk} are damping coefficients.

The perturbation (diffraction) forces and the torque are determined as follows:

$$F_{ej} = -\rho_2\eta_0 \int_L [i\omega(\Phi_0^{(2)} + \Phi_4^{(2)}) + V\nabla(\Phi_0^{(2)} + \Phi_4^{(2)})]n_j\,dl. \tag{1.12}$$

Since

$$\int_L V\nabla\Phi_k^{(2)} n_j\,dl = -U\int_L m_j \Phi_k^{(2)}\,dl,$$

by using the boundary conditions on the body (1.8) relations (1.11), (1.12) can also be represented in the form

$$\tau_{jk} = \rho_2 \int_L \frac{\partial\Phi_j^{(2)*}}{\partial n}\Phi_k\,dl,\ F_{ej} = \rho_2 \int_L \frac{\partial\Phi_j^{(2)*}}{\partial n}(\Phi_0^{(2)} + \Phi_4^{(2)})\,dl$$

(the asterisk denotes complex conjugate).

An effective numerical method of solving these problems in a homogeneous fluid is the Hybrid Finite Element Method (HFEM) [4, 8]. In this method the velocity potential for the lower layer is represented by the finite element method in a narrow region surrounding the body and by boundary integral equations in the exterior region. The HFEM combines the advantages of both methods; the flow behavior far from the body is reflected in the Green's function, while the choice of simple rectangular geometry of the exterior contour makes it possible to carry out the surface integration quite accurately. Note that in the HFEM one avoids the calculation of second derivatives of the stationary potential in boundary conditions (1.8). This method can be used for bodies of complex shape, systems of bodies, and can also be extended to the case of a stratified fluid, in which the density variation occurs only at horizons located above or below the submerged body.

A stationary load for an elliptic contour in a two-layered unbounded fluid was treated by the HFEM in [10], and perturbation forces during shaking without motion (U = 0) in a two-layered fluid, bounded from above either by a solid cover or by a free surface, were treated in [11].

2. The Green's Function. To use the HFEM it is necessary to determine the Green's function $G^{(s)}(x, y, \eta, \xi)$, satisfying the equations

$$\Delta G^{(1)} = 0 \ (0 < y < H),\ \Delta G^{(2)} = 2\pi\delta(x - \xi, y - \eta)\ (y < 0)$$

with boundary conditions similar to (1.5)-(1.7). The solution for $G^{(2)}$ is

$$G^{(2)} = \ln(rr_1) + 2(1 + \varepsilon)\mathrm{pv}\int_0^\infty \frac{F(k)}{kD(k)}e^{k(y+\eta)} \times \{[(U^2k^2 - \omega^2)^2 - (U^2k^2 + \omega^2)\Omega^2(k)]\cos k(x - \xi)$$

$$+ 2i\omega kU\Omega^2(k)\sin k(x - \xi)\}dk + \pi\{\alpha_1\exp[k_1(y + \eta - i(x - \xi))] - \alpha_2\exp[k_2(y + \eta - i(x - \xi))] \tag{2.1}$$

$$- \alpha_3\exp[k_3(y + \eta + i(x - \xi))] + \alpha_4\exp[k_4(y + \eta + i(x - \xi))]\}.$$

Here the symbol pv denotes an integral in the principal value sense,

$$r^2 = (x - \xi)^2 + (y - \eta)^2;\ r_1^2 = (x - \xi)^2 + (y + \eta)^2;$$

$$F(k) = (1 + \varepsilon + \mathrm{cth}kH)^{-1};\ D = D_1D_2D_3D_4;$$

$$D_{1,2}(k) = Uk + \omega \mp \Omega(k);\ D_{3,4}(k) = Uk - \omega \mp \Omega(k);$$

the function $\Omega(k)$ is determined by the dispersion relation (1.2), $\alpha_s = i(1 + \varepsilon)\Omega(k_s)F(k_s)/2k_s[U - \gamma c_g(k_s)]$ ($\gamma = 1$ for s = 1, 3, $\gamma = -1$ for s = 4); and $c_g(k_s) = d\Omega/dk\,|_{k=k_s}$ is the group velocity of wave k_s. The equation $D_1(k) = 0$ has two simple roots (k_1, k_2) with simultaneous satisfaction of the conditions

$$U < U_c,\ \omega < \omega_c, \tag{2.2}$$

where $U_c = \sqrt{g\varepsilon H}$ is the critical velocity for the stationary problem in the two-layered fluid considered, and $\omega_c = \Omega(k_c) - Uk_c$ is determined after solving the equation $c_g(k_c) = U$. When $\omega = \omega_c$ both roots coalesce, and no real roots exist when conditions (2.2) are not satisfied. The equation $D_2(k) = 0$ has no real roots, while the equations $D_3(k) = 0$ and $D_4(k) = 0$ always have one real root (k_3 and k_4, respectively). In the presence of all four roots they are arranged in the following order: $0 < k_4 < k_2 < k_1 < k_3$. In the moving coordinate system the k_1 and k_2 waves have positive phase velocities, and therefore they move to the right, but only the k_2 wave has a positive group velocity and propagates ahead of the body. The k_3 and k_4 waves have negative phase and group velocities and propagate behind the body to the left. In the fixed coordinate system the k_1, k_2, k_3 waves move to the right, and the k_4 wave — to the left. If conditions (2.2) are not satisfied, the terms including k_1 and k_2 in (2.1) must be removed (for more detail see [5, 8, 12]).

In the limiting case of an infinitely large upper layer ($H \to \infty$) the Green's function (2.1) is more conveniently written in the form

$$G^{(2)} = \ln r - \frac{\varepsilon}{2 + \varepsilon}\ln r_1 + 2\bar{g}\frac{1 + \varepsilon}{2 + \varepsilon}\mathrm{pv}\int_0 \frac{dk}{D(k)}e^{k(y+\eta)}$$

$$\times [(U^2k^2 + \omega^2 - \bar{g}k)\cos k(x - \xi) + 4i\omega Uk\sin k(x - \xi)] + i\pi(1 + \varepsilon)\{[e^{k_1(y+\eta-i(x-\xi))} \tag{2.3}$$

$$+ e^{k_2(y+\eta-i(x-\xi))}]/\sqrt{1 - 4\tau} + [e^{k_4(y+\eta+i(x-\xi))} - e^{k_3(y+\eta+i(x-\xi))}]/\sqrt{1 + 4\tau}\}/(2 + \varepsilon),$$

where $\tau = \omega U/\bar{g}$; $\bar{g} = \varepsilon g/(2 + \varepsilon)$, and

$$k_{1,2} = \frac{\bar{g}}{2U^2}(1 - 2\tau \pm \sqrt{1 - 4\tau});\ k_{3,4} = \frac{\bar{g}}{2U^2}(1 + 2\tau \pm \sqrt{1 + 4\tau}).$$

In this case $\omega_c = 0.25\bar{g}/U$. In the limit $\varepsilon \to \infty$ Eq. (2.3) provides the Green's function for an infinitely deep homogeneous fluid with a free surface, which was investigated in detail in [1, 4-7]. Computational algorithms for its calculation were given in [6]. For a homogeneous fluid of finite depth the Green's function for the problem under consideration was determined in [8].

In calculating the far field characteristics it is sufficient to restrict the discussion to Green's function values at $x - \xi \to \pm\infty$. For the Green's function (2.1) at $x - \xi \to \infty$

$$G^{(2)} \approx -2\pi\alpha_2\exp[k_2(y + \eta - i(x - \xi))],$$

and at $x - \xi \to -\infty$

$$G^{(2)} \approx 2\pi\{\alpha_1\exp[k_1(y + \eta - i(x - \xi))] - \alpha_3\exp[k_3(y + \eta + i(x - \xi))]$$

$$+ \alpha_4\exp[k_4(y + \eta + i(x - \xi))]\}.$$

Consequently, using (1.1) and the Green's equation, one obtains for $x \to \infty$

$$\Phi_j^{(s)} = A_{2j}\varphi_0^{(s)}(k_2)e^{-ik_2x} \; (j = \overline{1,4}); \tag{2.4}$$

and for $x \to -\infty$

$$\Phi_j^{(s)} = A_{1j}\varphi_0^{(s)}(k_1)e^{-ik_1x} + A_{3j}\varphi_0^{(s)}(k_3)e^{ik_3x} + A_{4j}\varphi_0^{(s)}(k_4)e^{ik_4x}, \tag{2.5}$$

where

$$A_{1j} = \alpha_1 e^{-k_1h}K_j^+(k_1); \; A_{2j} = -\alpha_2 e^{-k_2h}K_j^+(k_2);$$
$$A_{3j} = -\alpha_3 e^{-k_3h}K_j^-(k_3); \; A_{4j} = \alpha_4 e^{-k_4h}K_j^-(k_4);$$

and $K_j^{\mp}(k_s)$ is the analog of the Kochin function

$$K_j^{\pm}(k) = \int_L e^{k(\eta \pm i\xi)}\left[\frac{\partial\Phi_j^{(2)}}{\partial n} - k\Phi_j^{(2)}(n_1 \pm in_2)\right]dl.$$

For the diffraction problem in the fixed coordinate system only the k_4 wave can be in the opposite direction, while k_1, k_2, k_3 can be codirectional, depending on the frequency of the incident wave. For low frequency values ($\omega < \omega_c$) the incident wave is the wave with $k_0 = k_2$, and then it becomes the wave with $k_0 = k_1$. For $U > \omega_0/k_0$ the apparent frequency ω becomes negative, but since this has no physical meaning the time factor in (1.3) must be replaced by $e^{-i\omega t}$, as $\omega = k_0U - \omega_0$. In this case one must also change the sign of the first term in (1.8), and the Green's functions in (2.1), (2.3) must be replaced by their complex conjugates. The physical condition $U > \omega_0/k_0$ implies that the cylinder overtakes the incident wave, and in the moving coordinate system the wave moves to the right, in which case $k_0 = k_3$.

Knowing $\Phi_j^{(s)}$, one can determine the vertical elevation ζ of the separation boundary:

$$\zeta = \frac{1}{\varepsilon g}\frac{\partial}{\partial t}[\Phi^{(1)} - (1 + \varepsilon)\Phi^{(2)}]_{y=0}.$$

Taking into account (2.4) and (2.5), the amplitudes of radiation and diffraction waves in the far field satisfy

$$\zeta_{sj} = k_sA_{sj}/\Omega(k_s). \tag{2.6}$$

3. An Approximate Solution. Versions of the Haskind–Newman relations were obtained in [5, 8] for radiation and diffraction problems in the presence of motion. Unlike the case without motion, there exist no symmetries of the radiation loading matrix, and one can only express the diagonal damping coefficients and the perturbing forces in terms of the far field characteristics.

In the two-layer fluid considered the relation for the diagonal damping coefficients is

$$\lambda_{jj} = \frac{\rho_2}{\omega(1 + \varepsilon)}(M_1|A_{1j}|^2 - M_2|A_{2j}|^2 - M_3|A_{3j}|^2 + M_4|A_{4j}|^2), \tag{3.1}$$

where

$$M_s = \Omega(k_s)F^2(k_s)[U - \gamma c_g(k_s)]/\varepsilon g \;\; (s = \overline{1,4}).$$

According to the wave properties $U - c_g(k_{1,3}) > 0$, $U - c_g(k_2) < 0$, $U + c_g(k_4) > 0$, and, consequently, the third term in (3.1) is always negative, while the remaining terms are always positive. This implies that under certain conditions the damping coordinate system can acquire negative values, while in the absence of motion these characteristics are always positive. Relation (3.1), corresponding to the energy conservation law, physically implies that the k_3 wave guarantees a wave energy flux directed toward the body, while for the other three waves the wave energy flux is directed away from the body. The occurrence of negative λ_{jj} values is possible when the k_3 becomes dominant.

The perturbation forces are expressed similarly to (3.1) in terms of the far field characteristics of the radiation and diffraction problems.

Relations (3.1) make it possible to obtain an approximate solution for λ_{jj} under the assumption of deep submersion under the separation boundary of the body. The stationary flow potential, as well as the radiation and diffraction potential, are determined for a homogeneous unbounded fluid, and without including an upper layer. In this case the nonleaking condition at the surface of the body and the damping condition far from it are satisfied.

Consider the elliptic contour $x^2/a^2 + (y + h)^2/b^2$, where a and b are, respectively, the major and minor ellipse semiaxes, and h is the submersion depth of its center under the separation surface. Using the solution [13] for the stationary potential and the results of [11], the analog of the Kochin function can be represented in the form

$$K_1^{\pm}(k) = \pm \frac{4\pi\omega b^2}{c} J_1(kc) + b\int_0^{2\pi} [(a\sin^2\theta - b\cos^2\theta)S_1^{\pm} + (a + b)(a^2\sin^2\theta - b^2\cos^2\theta)S_2^{\pm}]d\theta, \tag{3.2}$$

$$K_2^{\pm}(k) = -\frac{i4\pi\omega a^2}{c} J_1(kc) - b(a + b)\int_0^{2\pi} \sin\theta\cos\theta(S_1^{\pm} + 2abS_2^{\pm})d\theta,$$

$$K_3^{\pm}(k) = 2\pi a[\pm 2b\omega J_2(kc) - U(a + b)J_1(kc)/c] + b(a + b)\int_0^{2\pi} \sin\theta\{aS_1^{\pm} + b[a^2 + (a^2 - b^2)\cos^2\theta]S_2^{\pm}\}d\theta.$$

Here $$S_1^{\pm}(k,\theta) = \frac{Uk(a\sin\theta \pm ib\cos\theta)}{a^2\sin^2\theta + b^2\cos^2\theta}\exp[k(b\sin\theta \pm ia\cos\theta)]; \quad S_2^{\pm}(k,\theta) = \frac{U\exp[k(b\sin\theta \pm ia\cos\theta)]}{(a^2\sin^2\theta + b^2\cos^2\theta)^2};$$

$c = \sqrt{a^2 - b^2}$; and J_n is the Bessel function of order n.

In the special case of a circular cylinder of radius b these expressions simplify substantially:

$$\{K_1^{\pm}(k), K_2^{\pm}(k), K_3^{\pm}(k)\} = 2\pi kb^2\{kU \pm \omega, -i(\omega \pm Uk), U\}.$$

The defect of this approximation is that $K_3^{\pm}$ does not vanish, despite the fact that rotational oscillations of a circular cylinder in an ideal fluid do not excite wave motion. This is, obviously, a consequence of the fact that in the stationary problem the given approximation provides a nonphysical result for the hydrodynamic torque [14].

For the diffraction problem, in calculating approximate $K_4^{\pm}$ values the results of [11] can be carried over completely: for an incident wave with $k_0 = k_1, k_2$

$$K_4^{+}(k) = 4\pi\sum_{n=1}^{\infty} na_n(k_0)a_n^{*}(k),$$

$$K_4^{-}(k) = -4\pi\sum_{n=1}^{\infty} na_n(k)b_n(k_0); \tag{3.3}$$

for an incident wave with $k_0 = k_3$

$$K_4^{+}(k) = -4\pi\sum_{n=1}^{\infty} na_n(k)b_n(k_0),$$

$$K_4^{-}(k) = 4\pi\sum_{n=1}^{\infty} na_n(k_0)a_n^{*}(k);$$

and for an incident wave with $k_0 = k_4$

$$K_4^{+}(k) = -4\pi\sum_{n=1}^{\infty} na_n^{*}(k)b_n^{*}(k_0),$$

$$K_4^{-}(k) = 4\pi\sum_{n=1}^{\infty} na_n^{*}(k_0)a_n(k),$$

where

In the special case of a circular cylinder $a_n(k) = (-i)^n(kb)^n/n!$, $b_n(k) = 0$, and for wave motion with $k_0 = k_1, k_2$

$$a_n(k) = (-i)^n\left(\frac{a + b}{a - b}\right)^{n/2} J_n(kc); \; b_n(k) = (-i)^n\left(\frac{a - b}{a + b}\right)^{n/2} J_n(kc).$$

while for wave motion with $k_0 = k_3, k_4$

$$K_4^{+}(k) = 4\pi b\sqrt{k_0k}I_1(2b\sqrt{k_0k}), \; K_4^{-}(k) = 0,$$

(I_1 is the modified Bessel function).

$$K_4^{+}(k) = 0, \; K_4^{-}(k) = 2\pi b\sqrt{k_0k}I_1(2b\sqrt{k_0k})$$

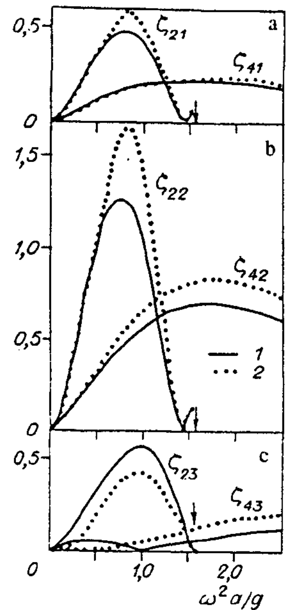

Fig. 1

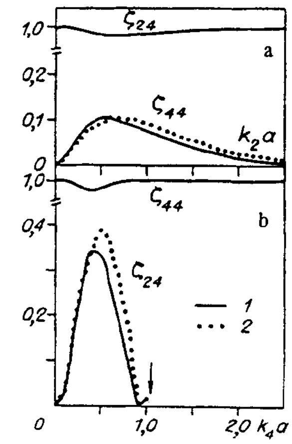

Fig. 2

Consequently, only one new wave is generated during wave scattering by a moving circular cylinder for arbitrary velocity values U. Only a reflected wave $k_2(k_1)$ is generated for wave motion $k_1(k_2)$. The incident wave k_3 (or k_4) is scattered into a k_3 and k_4 wave without the appearance of reflected waves. This is a consequence of the fact that there exists no reflection during wave diffraction by a circular cylinder with $U = 0$. A more detailed comparison of these approximate results with the complete solutions of the radiation and diffraction problems for a circular cylinder under the free surface of a homogeneous fluid was presented in [1].

A comparison of the approximate solution (3.2) with the numerical solution of the original problem for the radiated wave amplitudes, generated during the motion of the elliptic contour under the free surface of a homogeneous fluid is given in Figs. 1a-c, respectively, for horizontal, vertical, and rotational oscillations relative to the cylinder center with $U/\sqrt{ga} = 0.2$, $b/a = 0.3$, $h = a + b$. Line 1 shows the results of [3], and line 2 — the approximate solution (2.6) with the use of the approximate solutions (3.2). The arrows indicate the critical frequency value $\omega_c = 0.25g/U$. For the given parameters of motion the amplitudes of the k_1 and k_2 waves are negligibly small.

The diffraction wave amplitudes at the free surface are shown in Figs. 2a,b for the same parameters of motion, respectively, for the opposite k_4 wave and for the codirectional k_2 wave. Line 1 shows the results of [3], and line 2 — the approximate solution (2.6) with account of (3.3) for reflected waves. The relative amplitude of the transient wave is always is always equal to 1 within the given approximation. For the given parameters of motion the scattering in the k_1 and k_3 waves is negligibly small. It is seen that the approximate solutions describe quite well the qualitative behavior of excited waves, but somewhat distort their quantitative values.

4. Numerical Solutions. The most complete calculations of the radiation problem for a circular cylinder, submerged under the free surface of a homogeneous fluid of infinite depth, were given in [4], where, along with plots are also given tabular values of damping coefficients for a number of values of the original parameters, as well as plots of the combined mass and damping coefficients of elliptic cylinders. The calculations were carried out with the HFEM. The solution of the similar problem by the boundary integral equation method was presented in [1-3] for circular and elliptic cylinders. The case of a homogeneous fluid of finite depth was treated in [8], where the HFEM was used to determine the radiation and diffraction loads for a circular cylinder, and the diffraction load for an elliptic cylinder placed under an angle of attack. The complete statement of the problem, presented in Section 1, was used in all studies mentioned above. The numerical results, obtained within a somewhat simpler statement, when a stationary potential was selected to solve the radiation and diffraction problems, and determined for an unbounded fluid, are given in [5].

The Timman–Newman relations [15] are satisfied for low motion velocities of bodies symmetric with respect to the vertical axis:

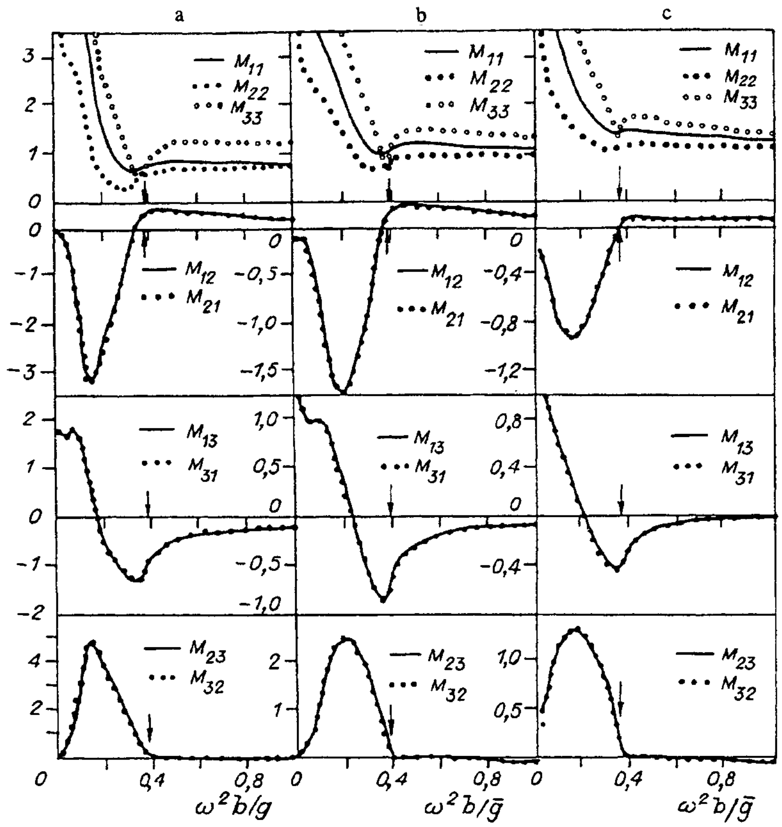

Fig. 3

$$\tau_{12} = -\tau_{21}, \tau_{23} = -\tau_{32}, \tau_{13} = \tau_{31}. \quad (4.1)$$

For $U \to \infty$, independently of the shape of the body $\tau_{jk} = \tau_{kj}^*$ [8, 15].

The numerical calculations of the combined mass and damping coefficients are given, respectively, in Figs. 3 and 4 for an elliptic cylinder, located under the free surface in a homogeneous fluid (a), under the separation surface in a two-layered unbounded fluid (b), and in a two-layered fluid with a bounded upper layer (c) with

$$a = h = 2b, \varepsilon = 0{,}03,\ H = b, U/\sqrt{\bar{g}b} = 0.4. \quad (4.2)$$

The number of elements in the HFEM equals 18. In Fig. 3 the combined mass coefficients are presented in dimensionless form

$$M_{jj} = \mu_{jj}/m_{jj} (j = \overline{1,3}),\ (M_{12}, M_{21}) = (\mu_{12}, -\mu_{21})/m_{11},$$
$$(M_{13}, M_{23}, M_{31}, M_{32}) = (\mu_{13}, \mu_{23}, \mu_{31}, -\mu_{32})/bm_{11},$$

where m_{jj} are the combined mass coefficients of an elliptic contour of an unbounded homogeneous fluid [13]:

$$m_{11} = \pi\rho_2 b^2,\ m_{22} = \pi\rho_2 a^2,\ m_{33} = \pi\rho_2 (a^2 - b^2)^2/8.$$

The dimensionless damping coefficients are given in Fig. 4 in the form

$$\Lambda_{jj} = \pi b\omega\lambda_{jj}/\bar{g}m_{jj}\ (j = \overline{1,3}),\ (\Lambda_{12}, \Lambda_{21}) = (\lambda_{12}, -\lambda_{21})\omega/\rho_2\bar{g}b,$$
$$(\Lambda_{13}, \Lambda_{31}, \Lambda_{23}, \Lambda_{32}) = (\lambda_{13}, \lambda_{31}, \lambda_{23}, -\lambda_{32})\omega/\rho_2\bar{g}b^2. \quad (4.3)$$

The upper portions of Fig. 4 show the approximate value of the diagonal damping coefficients $\bar{\Lambda}_{jj}$, obtained by using (3.1), (3.2), and values rendered dimensionless similarly to (4.3). It is seen that this approximation provides a quite crude representation for the damping coefficients, particularly for Λ_{33}. The crosses indicate the values of $\omega_c^2/\bar{g} = 0.3906$ (Figs.

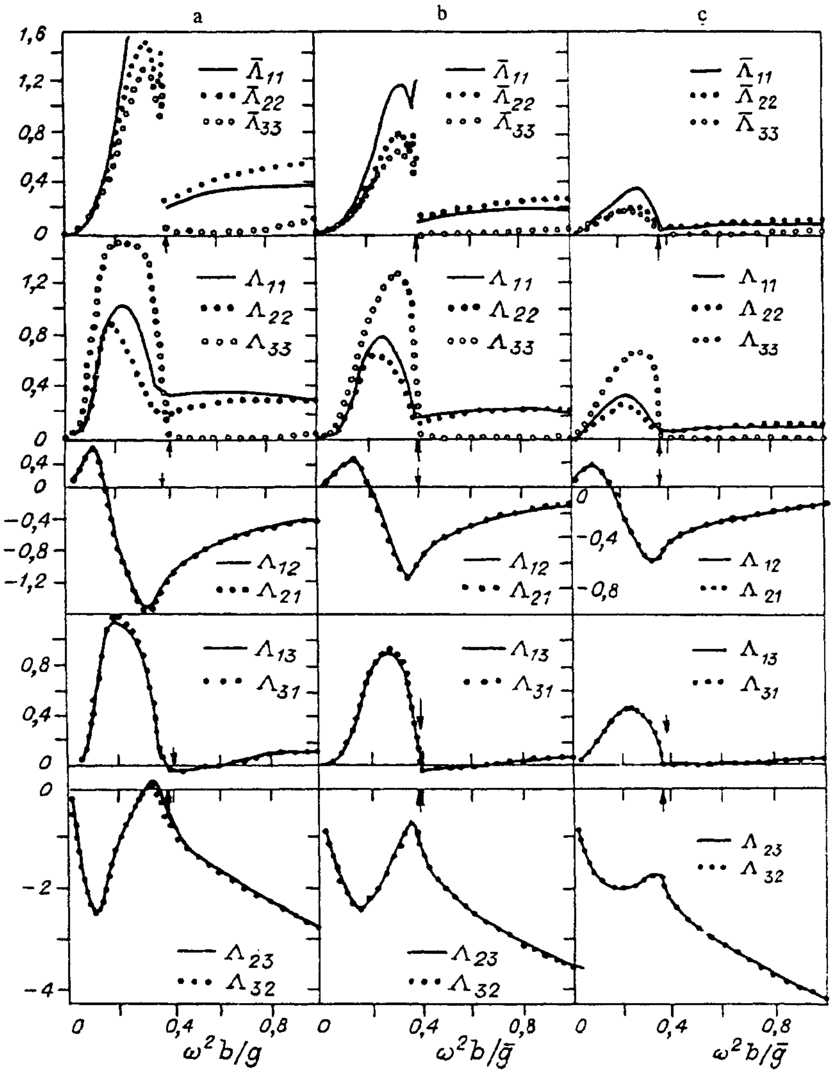
a
b
c
ω²b/g
ω²b/ḡ
ω²b/ḡ

Fig. 4

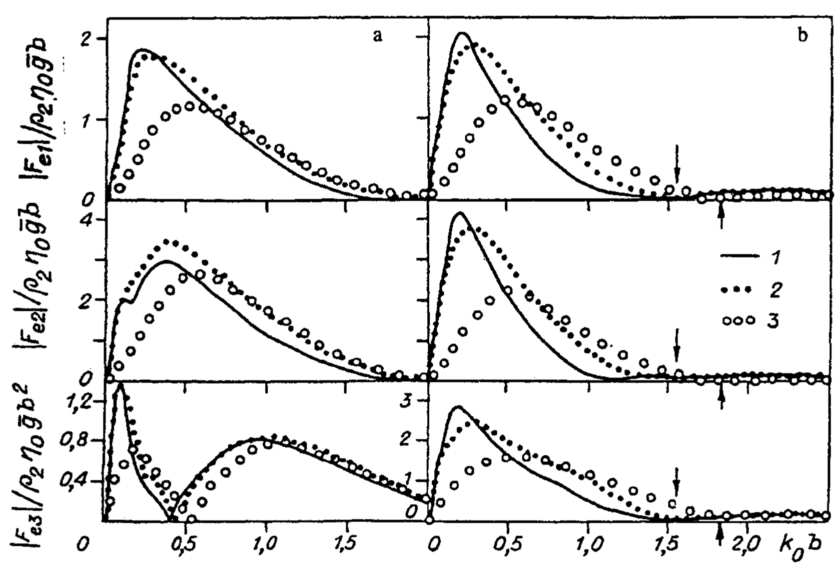
a
b
|Fe1|/ρ2η0ḡb
|Fe2|/ρ2η0ḡb
|Fe3|/ρ2η0ḡb²
1
2
3
k0b

Fig. 5

3a,b, 4a,b) and 0.3642 (Figs. 3c, 4c). For the given velocity of motion relations (4.1) are well satisfied. The effect of the velocity of motion on the radiation loading is most important at low frequencies.

The numerical results for diffraction loading on an elliptic contour at the same values of the original parameters of (4.2) are given in Fig. 5 for incident opposite (a) and codirectional (b) waves, while lines 1-3 corresponds to a homogeneous fluid with a free surface, and a two-layered fluid with an unbounded and bounded upper layer. The arrows in Figs. 5, 6 show the wave number values $k_0 b = 1.5626$ and 1.8277, corresponding to the critical frequencies ω_c for the unbounded and bounded upper layer. At these frequencies the velocity of motion coincides with the group velocity of the incident codirectional waves, and a sharp reduction occurs in the diffraction loads. Unlike the case without motion, even for a symmetric body the codirectional and opposite waves provide different perturbation forces. The results of the approximate solution, obtained under the assumption of a deeply submerged body, since, as in the case of the damping coefficients, they provide only a crude estimate. Figures 4a-c of [11] show values of the perturbation forces for the same geometric parameters of (4.2) and $U = 0$. The effect of motion is manifested most strongly on the torque value.

The calculations presented in Figs. 4, 5 refer to a low velocity of motion, practically not generating any substantial surface and internal waves (compare with the wave resistance characteristics in Figs. 3, 4 of [10]). However, even small stationary wave motions affect substantially the radiation and diffraction loading.

The studies performed make it possible to estimate the effect of stratification on the hydrodynamic characteristics of shaking a cylindrical body in the presence of motion.

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